p = propagation of the polymerization
t = termination of the polymerization

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Finite Amplitude Equilibrium Waves on the Surface of Nonvertical Falling Films

Experimental data on the frequency and wavelength of finite amplitude equilibrium waves on the surface of nonvertical laminar falling films of mineral oil at moderate Reynolds numbers are correlated successfully on the basis that the frequency of the wave remains constant during the course of its amplification.

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SCOPE

In many practical situations, one has to predict the properties of finite amplitude equilibrium waves on the surface of nonvertical falling films. In spite of many theoretical and experimental investigations that have been conducted on wavy flow in the past, there are many situations where these results are not useful. For example, since the waves are finite in size, the predictions of linear stability theories are not applicable. Also, the results of few nonlinear theories generally are restricted to vertical films and to limiting conditions not necessarily valid in a practical situation. Moreover, most of the experimental investigations have been conducted on vertical films, and not much data are available on the wave properties at small inclination angles. For this reason, an experimental work was undertaken to measure the properties of naturally occurring finite amplitude equilibrium waves on the surface of laminar films at inclination angles considerably different than 90 deg. In this paper, the results that were obtained for wave frequency and wavelength are presented. Since the frequency is the only wave property that is conserved at all the stages of growth, it may be used as a link between the properties of finite amplitude waves and the infinitesimal disturbance at the initial stage of growth. For this reason, data on wave frequency, rarely measured in the past, are used to correlate the properties of finite amplitude waves with the parameters furnished by linear stability theories.

CONCLUSIONS AND SIGNIFICANCE

For a given flow condition, the frequency of most highly amplified waves on falling films remains constant in all the stages of growth, starting from the initial state where the wave is in the form of an infinitesimal disturbance to the final state of equilibrium in which the wave attains a finite amplitude and constant properties. Therefore, the frequency of the wave can be used as a means to link the properties of finite amplitude waves to the properties of the disturbances at the initial stages of growth as predicted by linear stability theories. By using this concept, the frequency number γ and the wave number α of most highly amplified equilibrium waves with finite amplitude measured at various Reynolds numbers and inclination angles were successfully correlated as $\gamma = aN^n$ and $\alpha = bN^m$, respectively, with $N = (6/5\ Re - \cot\beta)/ReWe$ given by Yih's (1963) linear stability analysis. Within the range of the Reynolds numbers and inclination angles that were used in the experiments ($1.6 < Re < 49.5$, $10.1 < \beta < 45.8$ for frequency determination, and $1.85 < Re < 34.2$, $20 < \beta < 41.7$ for wavelength determination), n and m were found to be fairly constant, with lower values of n and m generally corresponding to the experiments that involved smaller Reynolds numbers. Values found for a and b consistently decreased with increasing β, indicating that the frequency number and wave number decrease at larger values of β. Considering the relative accuracy of the experimental procedure and the consistency of the data, the correlations presented in Figures 3 and 4 of this paper can be used for predicting the frequency and wavelength of finite amplitude equilibrium waves within the range of conditions studied in this work.

Finite amplitude equilibrium waves are frequently observed on the surface of falling liquid films. The prediction of the properties of these waves, namely, their wavelength, frequency, and amplitude, is needed for the determination of the effect of wavy flow on the rate of heat and mass transfer (Ruckenstein and Berbente, 1968; Javdani, 1974). For this reason, wavy flows have been the subject of many theoretical and experimental investigations in the past. Although theoretical treatments have provided useful information on the mechanism of wave formation and have identified the dominant parameters that are involved, they have not been able to provide accurate results for predicting the characteristics of the finite amplitude waves. This is especially true with the results of the linear theories (Yih, 1963; Krantz and Goren, 1971; Anshus, 1972; Marschall and Salazar, 1974) that are only valid during the initial stages of growth and cannot be expected to apply to the waves in a real situation. In a real situation, one is not dealing with an infinitesimal wave, nor with a growing wave, but with a most highly amplified wave that has already passed the stages of growth and reached a state of equilibrium with finite amplitude. In such an equilibrium state, the properties of the waves remain constant with time and distance but change with flow conditions, physical properties of the liquid, and also with the inclination angle of the flow.

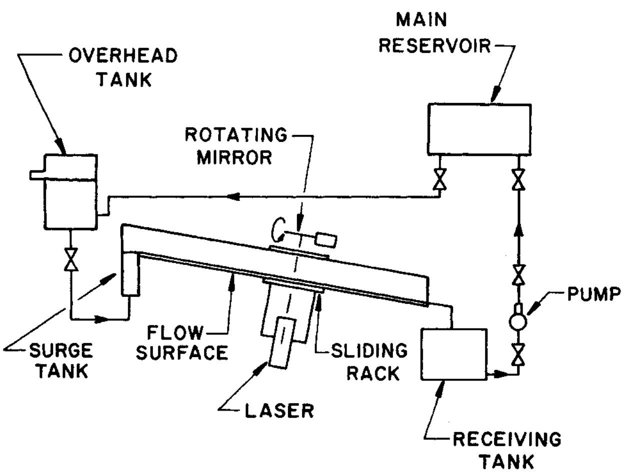

Fig. 1. Schematic representation of the apparatus.

There have been few attempts aimed at predicting the properties of these waves through numerical or approximate solutions to the nonlinear equations of motion (Nakaya and Takaki, 1967; Lin, 1971; Javdani and Goren, 1972). The results have shown relative success. However, as always, the numerical solutions lack the generality, and the approximate solutions include many simplifying assumptions that are not necessarily valid in practical situations. Moreover, to simplify the method of solution, in almost all of the nonlinear treatments the effect of the angle of inclination of the flow on the wave properties has been overlooked, making the results inapplicable for nonvertical flows. As a matter of fact, even the experimental measurements have been taken on vertical or near vertical films (Tailby and Potalski, 1960; Krantz and Goren, 1971), and not much experimental data are available on the effect of inclination angle on the properties of finite size waves. Thus, there is a need for experimental correlations that would predict the properties of these waves at different angles of inclination. This will be the subject of this paper in which data will be presented on the wavelength and frequency of finite amplitude equilibrium waves resulting from the amplification of natural disturbances on nonvertical falling films. It should be noted that so far not much information has been reported on the frequency of the waves, despite the fact that the frequency is the only property of the wave that remains constant during the course of its amplification. Actually, for this reason, the wave frequency can be used to link the properties of finite amplitude waves to the results of the theories that are valid for infinitesimal waves. In this paper, this unique feature will be used to correlate the properties of finite amplitude waves.

EXPERIMENTAL APPARATUS AND PROCEDURE

The experimental apparatus, shown schematically in Figure 1, consisted of an inclined channel 150 cm long and 20 cm wide, with its inclination angle with respect to horizontal variable from 0 to approximately 50 deg. The bottom of the channel was a flat glass plate, 5.7 mm thick and used as the flow surface. The sidewalls were made out of Plexiglass 2.5 cm thick and the top was covered by another glass plate, thus protecting the flow of the liquid film from possible connective currents in the laboratory. The liquid used was Duke mineral oil (μ = 33.6 centipoise, ρ = 0.884 g/cm, σ = 29.5 dyne/cm, at 25°C) furnished by Pars Oil Refinery. This liquid entered from the main reservoir into a constant level overhead tank from which it was fed onto the flow surface through a surge tank. The flow rate was adjusted by a control valve on the line carrying the oil from the overhead tank into the surge tank. The overflow of the liquid in the surge tank formed a thin film of liquid on the inclined flow surface on which the wave properties were measured. The liquid film leaving from and end of the channel was collected and returned to the main reservoir. With the help of leveling screws at the bottom of the equipment, the flow surface was adjusted so that a completely two-dimensional laminar flow was obtained in all of the experiments.

Naturally occurring, highly amplified waves were observed on the surface of the liquid film. Wave frequencies were measured by using a laser-mirror assembly that could be positioned at any location along the film. A schematic representation of the scheme is shown in Figure 2. A thin laser beam was passed through the film perpendicular to the direction of the flow. The beam refracted upon emerging from the free interface, with the amount of refraction depending on the refractive index of the oil and the slope of the surface. The emerging beam fell on a flat mirror rotating with its axis parallel to the flow, from which it was reflected onto a screen, producing a periodical pattern which was photographed. It can be shown that the wave frequency f can be obtained from this photographed pattern by using the following equation:

$$f = \frac{360r}{\tan^{-1}\dfrac{\Lambda}{2d}} \tag{1}$$

In this equation Λ is the wavelength of the photographed pattern, d is the distance between the screen and the mirror, and r is the angular speed of the mirror in revolutions per unit time. With this technique, the frequencies of the most highly amplified waves were measured for seven values of β ranging from 10.1 to 45.8 deg. In this range, the Reynolds numbers were normally large compared to unity and varied from 1.6 to 49.5. The wavelengths of the most highly amplified waves were measured by simply photographing the surface of the film.

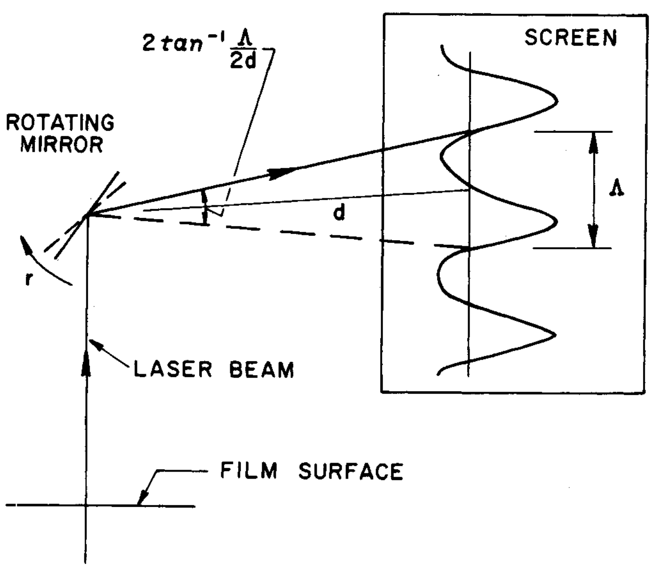

Fig. 2. Schematic representation of frequency measuring scheme.

Illuminating the surface would produce reflections from the wave crests that were recorded as bright spots on the photograph, thus providing the wavelength. Wavelengths were measured for four values of β ranging from 20 to 41.7 deg, with Reynolds numbers varying from 1.85 to 34.2. No measurements were taken for β < 20 deg because the direct photography of the flow surface did not produce an accurate recording of the wavelength at low inclination angles. Wave frequencies and wavelengths were measured far downstream from the line of inception, where the waves had grown into finite size and had reached a state of equilibrium. This is especially important in wavelength measurements since, unlike frequency, this property is more susceptible to change along the film. Study of the photographs taken shows that the bright spots representing consecutive waves were equally apart from each other, indicating that the wavelength had indeed reached its constant value at the location where the measurement was taken. Typical data on the constancy of the wavelength at distances far from the entrance are shown in Figure 3.

CORRELATION OF DATA

Frequently the results of linear stability theories are used to correlate the experimental data on wave characteristics. However, as was discussed earlier, the applicability of the results of these theories for predicting the properties of finite amplitude waves is questionable; thus, correlating parameters can not be obtained directly from them. Nevertheless, a correlation can be derived from the results of these theories if the wave frequency is used as a link between the properties of the finite amplitude waves and the properties of the infiniterminal disturbances at the initial stages of growth. At the onset of instability in the flow, among the natural disturbances with different frequencies, a single frequency will have a maximum growth rate, and the disturbances with other frequencies will either decay or be suppressed by the disturbance with the maximum growth rate. This same disturbance will grow and eventually reach the final state of equilibrium that is observed on the surface of the film. During the course of amplification, the properties of this most highly amplified wave changes, with the exception of its frequency which stays constant. The constancy of the frequency of the most highly amplified waves has been verified experimentally by Krantz (1968). For this reason, the frequency of the waves that are observed is the same as their frequency at their initial stage of growth and therefore can be predicted from the results of linear theories.

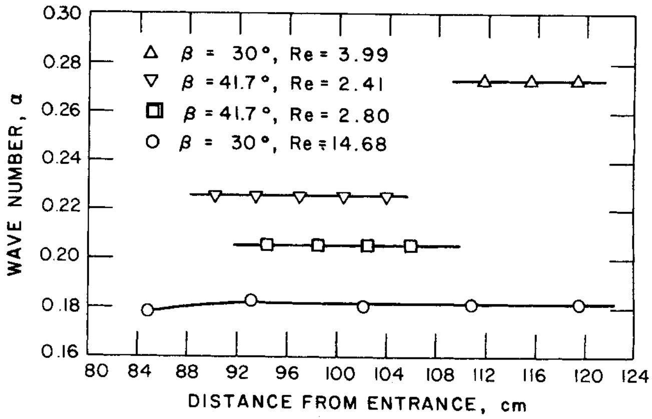

Fig. 3. Variation of wave number along the film at long distances from entrance.

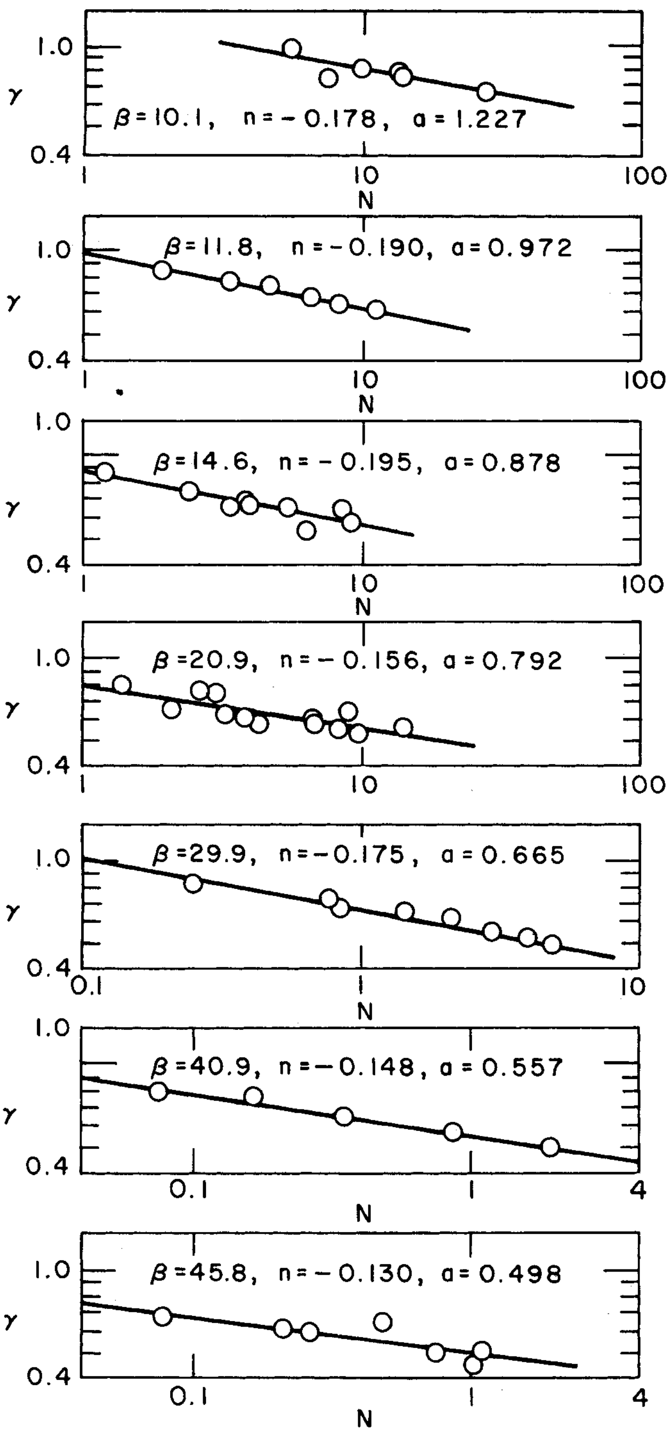

Fig. 4. Frequency number for different inclination angles.

A simple relationship, $f = u_w/\lambda$, exists between the frequency f, the wave velocity u_w, and the wavelength λ. Writing this relationship in dimensionless form, we will have

$$\gamma = \alpha C_r \tag{2}$$

where $\gamma = 2\pi\bar{h}f/\bar{u}$ is the frequency number, $\alpha = 2\pi\bar{h}/\lambda$ is the wave number, and $C_r = u_w/\bar{u}$ is the dimensionless velocity of the wave. According to the argument presented above, for the same flow conditions (same $\bar{u}$ and $\bar{h}$) the frequency number should be equal to the product αC_r given by the linear stability theory. Thus

$$\gamma = (\alpha C_r)_L \tag{3}$$

where the subscript L denotes that the quantity is derived from linear stability theory. Unfortunately, even the linear theories do not provide closed-form solutions for the wave properties under all conditions, and rigorous results exist for limiting cases only. For example, when the wave number is small and the Reynolds number is in the order of unity, the linear stability theory of Yih (1963) shows that

$$\alpha \sim N^{1/2} \tag{4}$$

where $N = \dfrac{6/5\,Re - \cot\beta}{ReWe}$, and

$$C_r = \text{constant} \tag{5}$$

Therefore, for this limiting condition Equation (3) becomes

$$\gamma \sim N^{1/2} \quad [Re = 0(1), \;\; \alpha << 1] \tag{6}$$

For other limiting conditions, asymptotic solutions to the Orr-Sommerfeld equation have been presented by Anshus (1972). According to Anshus, for large Reynolds numbers the variation of the wave number and wave velocity of the infinitesimal disturbances with maximum rate of growth with Reynolds number can be described by the following asymptotic forms:

$$\alpha \sim Re^{1/3} \tag{7}$$

$$C_r = 3/2 + 0(Re^{-2/3}) \tag{8}$$

The magnitude of the second term on the right-hand side of Equation (8) is not necessarily small in comparison with the first term, especially at moderate Reynolds numbers. This means that under this condition the frequency number γ will be equal to the sum of two terms that are proportional to $Re^{1/3}$ and $Re^{-1/3}$, respectively, with each of them contributing significantly. In order to include the effect of the inclination angle and also to maintain consistency with Equation (6), we should express γ in terms of N rather than Re. Actually, for a given fluid, We is not an independent parameter since it is equal to $\xi Re^{-5/3}$, where $\xi = (3/g)^{1/3}\,\sigma/\rho\nu^{4/3}$ is the surface tension parameter and is constant for a given fluid. Therefore, it is easy to show that N will vary as $Re^{5/3}$ as the Reynolds number becomes large, provided that $\cot\beta << Re$ and that γ consists of two terms proportional to $N^{1/5}$ and $N^{-1/5}$, respectively. Therefore, for all practical purposes and without introducing any significant error, we choose N as the single parameter to correlate the experimental data, and in order to be consistent with the rigorous result of Equation (6), we look for a correlation of the form

$$\gamma = aN^n \tag{9}$$

where n may have different values depending on the conditions of the experiment, and a is the proportionality constant that may depend on β. Accordingly, the frequencies measured with the technique described above for seven angles of inclination were plotted in Figure 4. For each inclination angle, best values of a and n were obtained by using the least-square fit of Equation (9). The results, in qualitative agreement with the argument presented above, are shown in Figure 4 also. It is seen that for lower angles of inclination, where the Reynolds numbers were large, n has its lowest values. At higher values of inclination angle, the experiments had to be stopped at lower Reynolds numbers (although still larger than unity) to keep the two dimensionality of the flow intact. Accordingly, the exponent n increases with β, showing a gradual change in the behavior of the frequency number and moving toward its assymptotic values at low Reynolds numbers. For some unknown reason, the results for $\beta = 20.9$ deg seem to be inconsistent with others. The results for the proportionality constant a show that they consistently decrease with inreasing values of β, indicating that the frequency number decreases with increasing the inclination angle.

Equation (9) can be used with Equation (2) to establish a correlation for the wave number of finite amplitude waves at different values of β. Previous theoretical work on finite amplitude wave velocities shows that, at least for vertical films, C_r attains a constant value (Lin, 1971; Javdani and Goren, 1972). The behavior of finite amplitude wave velocity at low angles of inclination is not clear. However, even if it is not to be constant, it is reasonable to assume that it depends on N, since N is the best combination that includes the effect of all the dominant groups, namely, Re, We, and β. For this reason, we expect that

$$\alpha = bN^m \tag{10}$$

Accordingly, the wavelength data were plotted for four values of β as shown in Figure 5, and satisfactory agreement was obtained. For each inclination angle, best values for b and m were calculated from the data by using the least-square fit of Equation (10). The results are shown in Figure 5 also. Values of m are found to be very close to n for each value of β, indicating the C_r is indeed a weak function of N. Again, except for the data at 20 deg, m is found to increase consistently with increasing values of β. This is again attributed to the fact that the experiments at higher values of β were generally terminated at lower values of Re in order to preserve the two dimensionality of the flow. Values of b are found to decrease consistently with increasing values of β, indicating that wave number decreases when the inclination angle is increased.

Since wave velocities were not measured directly, there will not be any attempt to present the results calculated from wave frequency and wavelength data. However, in general, such an attempt would result in a correlation for C_r as a weak function of N.

The results presented here for the wave numbers show marked differences with the data obtained by other investigators on vertical columns (Tailby and Portalski, 1960; Krantz and Goren, 1971). The plot of these data against the parameter N, which reduces to We^{-1} for vertical columns, has shown that m generally has positive values. One explanation for this difference is that in these experiments either the measurements have been taken on or near the line of wave inception, where the waves are still growing, or the corresponding Reynolds numbers were relatively low, for which case m has to approach 0.5 as given by Yih's theory. Certainly, additional measurements at higher angles of inclination ($41.7 < \beta < 90$) are needed to discover if a gradual increase in the value of m can be obtained. Actually, the correlations in Figure 5 show that such a trend exists. However, as was mentioned, this trend seems to be caused by the gradual reduction in the range of the Reynolds numbers corresponding to each inclination angle. It may happen that with a further increase of the inclination angles (that would correspond to a gradual decrease in the values of Reynolds numbers that can be maintained), a gradual change in m (and n) would eventually produce positive values for m (and n). This remains to be seen. Nevertheless, considering the relatively accurate techniques that were used for frequency and wavelength measurements and the consistency of the data, we can conclude that the correlations presented in Figures 4 and 5 can be used for the prediction of the properties of finite amplitude equilibrium waves within the range of conditions studied in this work.

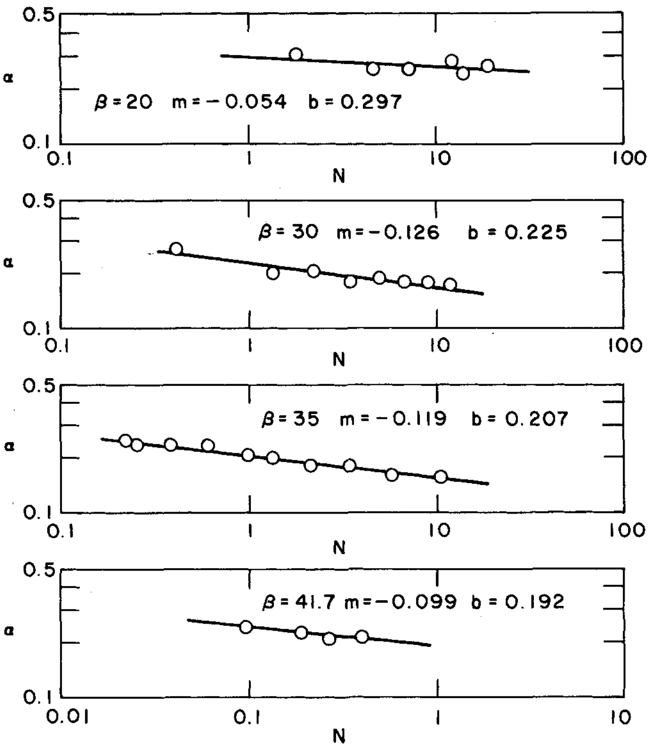

Fig. 5. Wave number for different inclination angles.

NOTATION

- a = proportionality constant for frequency number, Equation (11)
- b = proportionality constant for wave number, Equation (12)
- C_r = dimensionless wave velocity = $u_w/\bar{u}$
- f = frequency
- $\bar{h}$ = average film thickness
- m = defined by Equation (12)
- n = defined by Equation (11)
- N = correlating parameter = $\dfrac{6/5\,Re - \cot\beta}{Re\,We}$
- Re = Reynolds number = $\bar{u}\bar{h}/\nu$
- $\bar{u}$ = average velocity of the liquid
- u_w = wave velocity
- We = Weber number = $\sigma/\rho\bar{h}\bar{u}^2$

Greek Letters

- α = wave number = $2\pi\bar{h}/\lambda$
- β = inclination angle
- γ = frequency number = $2\pi\bar{h}f/\bar{u}$
- λ = wavelength
- ν = kinematic viscosity of the liquid
- ρ = density of the liquid
- σ = surface tension of the liquid

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On the Dynamics of Fluid Interfaces

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SCOPE

Under conditions of equilibrium, the property of interfacial tension is sufficient to completely describe fluid-fluid interfaces. However, it has been realized that under dynamic conditions, interfaces exhibit properties that are quite unique. Consequently, in recent years, fluid interfaces in motion have excited the attention of a large number of researchers in a variety of problems of far-reaching interest physically. To mention a few of these, one can list the phenomenon of capillarity in classical hydrodynamics, the damping of water waves by films of oil, the chemistry of surface films, the stability of foams and emulsions, the behavior of drops and bubbles, the enhancement of transfer across interfaces, and a wide variety of other problems.

The unique nature of these dynamic interfacial properties has even led to the characterization of interfaces as a distinct phase. Interfacial mechanics deals with the resistance offered by interfaces to viscous and elastic forces and consequent dynamic properties. In recent years, several mathematical models have been proposed to describe the dynamics of an interface, and these usually incorporate properties such as dilational viscosity and elasticity and shear viscosity and elasticity. Experimental measurement of these and their verification seem very scanty.

The objective of this work is to theoretically and experimentally analyze an interface in motion with a view to throwing a little more light on these properties. The system of dispersed phases oscillating in a continuous medium is chosen to provide a dynamic interface. From the existing theories of interfacial mechanics and the basic theory of oscillating dispersed phases in a continuous medium, the decay and frequency of these oscillations are computed as functions of the dynamic interfacial properties. An experimental system is set up, and the decay and frequency of liquid drops and gas cavities of varying radii are measured. On the belief that these dynamic properties are altered by the presence of surface active agents, the system was maintained very clean, and experiments were performed with triple distilled water, tap water, and accurate aqueous solutions of ionic and nonionic surfactants. A comparison with the theoretical predictions was made in an attempt to estimate some of the interfacial properties.

CONCLUSIONS AND SIGNIFICANCE

The theoretical computations were made by assigning values to the interfacial properties reported in the literature. From the general system of disperse phases oscillating in a continuous medium, two simplified cases were chosen. These were the case of a drop of liquid oscillating in a gas and a gas cavity oscillating in a liquid. The size range of the drops and cavities studied were from 0.8 to 1.5 mm. This range was chosen from experimental consideration. Sizes below 0.8 mm were not studied because of the very high frequency that they exhibit, making measurements inaccurate, and sizes beyond 1.5 mm exhibit deformation from the spherical shape, making the theory inaccurate. Within this size range, it was found that the interfacial viscosities had little effect on the decay and frequency. The dominant interfacial properties in this range are the elasticities. It was found that the dilational elasticity dominated in the case of the drop, and the shear

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